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ABSTRACT

This paper describes a review and synthesis of research on algebra. The purpose was to identify findings that may prove useful to teachers, supervisors, and administrators. Over 300 studies were identified and categorized in terms of classroom instructional practices, cognitive factors, attitudes and affective factors, problem solving, calculators and computers, and testing and prediction. Then the studies in each category were reviewed to identify worthwhile ideas for school practitioners. Thirty-nine key ideas were selected from about 150 studies for inclusion in the final review. A sample from each category is given; the complete monograph will be published by the National Council of Teachers of Mathematics. Three caveats, a cost analysis of research, and future types of reviews are outlined. (MNS)

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A REVIEW AND SYNTHESIS OF TWELVE YEARS OF RESEARCH RELATED TO SECONDARY SCHOOL INSTRUCTION IN ALGEBRA

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A REVIEW AND SYNTHESIS OF TWELVE YEARS OF RESEARCH RELATED TO SECONDARY SCHOOL INSTRUCTION IN ALGEBRA

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Introduction

There have been literally hundreds of research studies completed in mathematics education during the twelve year period of 1970 through 1981. The results of these studies are published as reports in a variety of journals ranging from the specialized Journal for Research in Mathematics Education through the more general American Educational Research Journal. In addition, many studies arise from doctoral dissertations and appear as abstracts in Dissertation Abstracts International. These journals and abstracts tend to reach specialists in mathematics education research but are not often found in the personal libraries of school practitioners: teachers, supervisors, principals, and superintendents. Because research results do not often reach the practitioner level, there is a serious loss of impact of research on classroom instruction. This is a great loss of effort and resources which should be remedied.

Purpose of this Paper

The purpose of this paper is to report the highlights of an

extensive study done to review and synthesize twelve years of research on instruction in secondary school algebra. The primary purpose of the study (Dessart and Suydam, to appear, 1983) was to identify "classroom ideas"; that is, clear, succinct statements of research findings devoid of technical details that may prove useful primarily to classroom teachers but also to supervisors, principals, and superintendents. These ideas will appear as part of a monograph entitled, Classroom Ideas from Research on Secondary School Mathematics, to be published by the National Council of Teachers of Mathematics (NCTM) during the summer or fall of 1983. The monograph will consist of two parts: one dealing with algebra instruction and a second dealing with geometry. Presumably, other parts covering junior high school grades and more advanced mathematics courses may be published at later dates, although there are no firm plans for this possibility at the present time.

Methodology of the Study

The methodology of the study consisted of several phases. In the first phase of the project all research studies on algebra instruction reported by Suydam and Weaver, 1971 through 1982, in their annual annotated bibliographies appearing in the Journal for Research in Mathematics Education were identified. It should be noted that the Suydam and Weaver bibliographies consist of titles systematically collected from over 50 journals. The bibliographies contain sections on research summaries, journal-published reports, and dissertation abstracts. In this search over 300 titles were identified dealing with instruction in algebra; 200 of these were dissertation abstracts

and about 100 were journal - reported articles.

In the second phase of the project, the 300 studies were partitioned into six categories that are listed below. The numbers in parentheses are the numbers of titles in each category.

1. Classroom Instructional Practices (88)
   (includes organizing for instruction, the algebra teacher in the classroom, and homework).
2. Cognitive Factors (63)
   (includes critical thinking, memory, intuition, creativity, and interactions).
3. Attitudes and Affective Factors (40)
   (includes attitudes of teachers, students, parents, and their interactions).
4. Problem Solving (42)
   (includes characteristics of problem solvers, heuristics, and instruction in problem solving).
5. Computers and Calculators (39)
   (includes computer programming, computer-augmented instruction, computer-assisted instruction, and hand-held calculators).
6. Testing and Prediction (36)
   (includes prediction of success, sex differences, and test-taking strategies).

In the third phase of the study, the research reports in each of the six basic categories were reviewed category by category. The primary goal was to identify "ideas"; that is, results which might prove useful to school practitioners. The emphasis was a pragmatic one; that is, a search for worthwhile "ideas". In this process three

criteria were used to select ideas. These three criteria are given below:

1. The idea should be useful to school practitioners.
2. The idea should be succinct, clear-cut and unencumbered by technical details and jargon.
3. The idea should be supported by valid research designs, including adequate sampling, statistical designs, and other research factors.

The last criterion was not accomplished by a rigorous examination of the details of studies because of time constraints. Much of the reported research had been carefully examined by journal referees so that its appearance in a published form was indicative of a reasonably adequate research design. Furthermore, one would hope the same type of scrutiny had been given doctoral dissertations.

The Ideas

Thirty-nine key ideas were selected from about 150 studies that were part of the written review (i.e., the formal list of references contains about 150 titles). The distribution of the 39 ideas is given below:

1. Classroom Instructional Practices 8
2. Cognitive Factors 8
3. Attitudes and Affective Factors 5
4. Problem Solving 7
5. Computers and Calculators 3
6. Testing and Prediction. 8

39

A sample of the ideas is given below with one idea selected from each of the six categories.

1. Classroom Instructional Practices.

   Idea: Should you make detailed comments on your students' homework papers? Most algebra teachers probably feel that the practice produces superior results over the practice of not making comments. Research indicates that detailed comments do not significantly improve student achievement when compared to not making comments. Teachers should interpret this finding with caution, as student attitudes may very well be bolstered by comments of praise or approval.

2. Cognitive Factors.

   Idea: Critical thinking skills, such as those needed for the processes of induction, deduction, making assumptions, and so forth, can be improved through the study of algebra. But research seems to indicate that such higher order cognitive skills are not often demanded of students in the classroom.

3. Attitudes and Affective Factors.

   Idea: Teachers tend to have more favorable attitudes toward "mathematics as a process" than do students; however, students seem to have more favorable attitudes toward the "role of mathematics in society" than do their teachers. These attitudinal differences probably reflect the most relevant concerns of both groups!

4. Problem Solving.

   Idea: Some algebra students prefer to "guess and check" in setting-up an equation to solve a verbal problem. Other students prefer to organize the problem data into tables from which the equation is derived. Teachers probably prefer the latter approach. Research shows that the "guess and check" procedure does have merit for initial learning experiences.

5. Computers and Calculators.

   Idea: Having algebra students program certain mathematical topics for the computer does not appear to necessarily enhance their achievements related to those topics. Perhaps, computer programming should be simply learned for its own utility and not because of some hoped-for by-products.

6. Testing, Competencies, and Predictions.

Idea: As a result of her research, Swafford stated: "Sex differences in achievement and attitudes that emerge in first-year algebra have little to do with algebra. Although this overall comparability of males and females after their first high school mathematics course is reassuring, the disparity in consumer exercises is cause for concern." (Swafford, 1980, p. 344)

The Monograph

The complete monograph will be published by the National Council of Teachers of Mathematics (NCTM) during the summer or fall of 1983. The monograph will include discussions of the highlights of each study including an indication of the classes used in the studies, the nature of instruments used, and a summary of findings. Such technical details as tables of data, statistical tests utilized, and other design details are not discussed unless they have significant purpose relevant to the discussion. The reason for such an approach is because it was felt that busy practitioners would not have time to read such details and, in fact, such details may discourage their reading of the monograph. The primary goal of the monograph is to be useful to school practitioners.

The 39 ideas that have been identified earlier are culminations of the discussions of the research. The ideas are highlighted throughout the volume by placing them in rectangles interspersed in the text material. The reader may easily peruse the volume, savoring the ideas by paging through the volume, stopping at certain rectangles to read more completely about an idea that captures his or her attention. In this sense the "boxed-ideas" serve a similar purpose as headlines in a newpaper.

Caveats

In a monograph of this kind designed to inform practitioners of research results, certain caveats or warnings are certainly in order. First of all, frequently research results in mathematics education appear to be "self-evident" to the reader. Perhaps, it is reassuring to the reader that some of these self-evident notions have been supported by research. Nevertheless, one must keep in mind that a researcher must have felt that the question of the self evidence was subject to speculation. This point-of-view should be respected as many self evident truths may not, in fact, stand the test of rigorous study and research.

Second, research findings are always tentative. There are no "absolute truths" in empirical research. The findings that may be appropriate for a particular set of students at a particular time and place may not be applicable to another set of students at another time and place. One must always respect the fact that the inference of findings from one sample of students to another is always subject to error.

Third, with these warnings in mind, practitioners should approach research findings with an experimental point of view. That is, practitioners should feel that if they wish to use these ideas then they should be willing to experiment. They should be willing to try the ideas with their students and judge for themselves their usefulness. Experimentation is far too important an activity to be left entirely to professional researchers. All practitioners should be researchers!

A Cost Analysis

As one contemplates that approximately 300 studies were completed in the area of algebra instruction during the period of 1970 through 1981, a natural question seems to arise concerning the cost of such research. In this section a simple cost analysis will be made to support the fact that the non-use of research findings by school practitioners represents a terrible waste of time and money.

Assuming that each study required at least six weeks of the total efforts of at least one person whose salary is $25,000 per year, it can be shown that each study would cost approximately $3,000 in time (a conservative estimate). If one would also add approximately $300 for miscellaneous costs, it seems reasonable that each study would cost conservatively $3,300-$3,500. Three hundred studies would then amount to approximately $1,000,000.

From these 300 studies, the author identified some 39 "ideas" which were highlighted for sharing with school practitioners. Considering that $1,000,000 generated 39 ideas, the cost per idea is in the neighborhood of $25,000! It seems that one cannot hesitate to conclude that not making use of findings from research available to school practitioners is unforgivably wasteful!

The Future

As has been expressed earlier in this paper, the main purpose of the study was to identify ideas that could prove useful to school practitioners. It is obviously hoped that the ideas are valid and reliable, but there is no claim made in this paper nor will there be a claim made in the published

monograph concerning the validity and reliability of these ideas. It has been suggested in the section on caveats that practitioners should experiment with the ideas to determine their usefulness. To the extent that these ideas provide a viable source of information for practitioners is the primary criterion that the work should be judged.

But for the future, this criterion of usefulness may not be enough. There are currently available meta-analyses of research: ways of cumulating research findings across studies in rigorous ways. These techniques are discussed in a variety of references including Meta-Analysis: Cumulating Research Findings Across Studies (Hunter, Schmidt, and Jackson, 1982). Hunter, Schmidt, and Jackson identified a number of methods for integrating results across studies. These include The Narrative Procedure, The Rating Method, Cumulation of p-values, and Meta-Analyses.

The Narrative Procedure is the most popular of the four methods. It consists largely of describing the results of studies without a statistical examination of results. The reviewer attempts to ascertain the preponderance of evidence and relies upon "a seat of the pants" judgment for reaching conclusions. One might also term this procedure the "one-man jury" method. It is probably reasonably efficient when the number of studies across a specific topic is small, say, less than 10-15. But the mental burden of summarizing large numbers of studies is perhaps too much for the average reviewer, so that more effective means of categorizing large numbers of studies are needed.

One such method is the Voting Method. In this procedure a tally is kept of the numbers of positive, negative, and neutral statistical significant findings on a particular topic. The winning conclusion is the one that accumulates the largest number of positive findings. The Voting Method

tends to favor conclusions that arise from positive findings because negative results often do not find their ways into publications.

A third procedure involves the cumulation of p-values across studies. An overall p-value is determined for the entire set of studies; much as if the entire set of studies had constituted a single study. This procedure produces a simplistic result, that is, an overall p-value but often times the magnitude of the effect is left unknown. However, there are statistical procedures for testing the statistical significance of the p-value and for estimating the effect sizes if the sample sizes are known (Hedges and Olkin, 1980).

The most powerful methods of cumulating research findings across studies fall under the rubric of Meta-Analyses. Two of these analyses, the Glass Method and the Schmidt-Hunter Method, place a strong emphasis upon the notion of effect size, which is the difference of the treatment and control means divided by the average standard deviation or the standard deviation of the control group. From these effect sizes, a mean and standard deviation of cumulated effect sizes are found. These prove to be useful measures of comparison.

In the Glass Method the variance of effect sizes is also studied at face value for substantive explanations (Hunter, Schmidt, and Jackson, p. 138). The Schmidt-Hunter Method analyzes the variance of effect sizes further to determine if they were due to various statistical artifacts; such as, sampling error, reliability of measures of independent and dependent variables, various range restrictions, instrument validity, and other factors.

These represent a few of the ways of organizing conclusions across studies. The Narrative Approach may very well remain the most popular

procedure, but Meta-Analyses offers more rigorous means of making judgments from many research reports.

Conclusion

In this paper a study involving a synthesis of 12 years of research on algebra instruction was reviewed. The purpose of the paper was described, the methodology of the study was discussed, a sample of "ideas" which are clear, short statements selected from the research for use by school practitioners was provided, the monograph was discussed, certain caveats were listed, a cost analysis to justify the necessity of using the ideas was included, and future types of reviews were outlined. It is hoped that this paper and the monograph to be published in 1983 will prove useful to school practitioners and educational researchers.

References

Dessart, D. J. and Suydam, M. N. Classroom Ideas from Research on Secondary School Mathematics. Reston, Virginia; NCTM, to appear, 1983.

Hedges, L. V. and Olkin, I. Vote-counting methods in research synthesis. Psychological Bulletin, 1980, 88, 359-369.

Hunter, J. E., Schmidt, F. L., and Jackson, G. B. Meta-Analysis: Cumulating Research Findings Across Studies. Beverly Hills, California: Sage, 1982.

Suydam, M. N. and Weaver, J. F. Research on mathematics education reported in 1970...1981. Journal for Research in Mathematics Education, 1971... 1982, Vol. 2 through 13, No. 4 of each volume.